Research Article

DCSI 07: 131 - 134 (2012)
Received: 28 April, 2012
Revised: 06 June, 2012
Accepted: 25 June, 2012
www.jdcsi.in

Viscous Fluid Model by Hele Shaw Method and Study of Dimension Calculations

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Abstract:

Viscous fluid model by using Hele Shaw method gives a fractal geometry pattern; it plays important role study of irregular shapes. Such shapes are found in many growth patterns and exhibit self similarity. These finger patterns are similar to the structures obtained in diffusion limited aggregation.

Using various viscous fluids in Hele Shaw cell, viscous fingering is studied. The patterns obtained are photographed, digitized and converted in to matrix for further processing. We find the resulting finger structure of Glycerol and Fractal dimensions are obtained by using box counting techniques. The change in viscous fingering pattern with different pressures is also studied and Fractal dimension D=1.6 for low pressure and 1.80 for High pressure is obtained.

Introduction:

Viscous fingering is a phenomenon in which less viscous fluid is injected into more viscous fluid under controlled condition which leads to development of fingered interface. When a fluid displaces another fluid with higher viscosity the interface is unstable and driving fluid intrudes into the viscous fluid in the form of “Fingers”, (Saffman and Taylor 1958), developed the theory of viscous fingering for situations where the fluids are immiscible and where the flow is described by Darcy’s law valid in porous media for each of the fluid separately. This theory also describes the flow of ordinary fluids in a two dimensional geometry given in Hele Shaw cell (B. B. Mandlebrot et. al. 1983 and David C. Caccia et. al. 1997). The Hele Shaw cell is a good example of the system where an interface is allowed to grow between two closely placed surfaces. The viscous fluid is placed in the cavity between the two plates and less viscous fluid is injected through a hole at the center. It has been shown that at low pressure the growth exhibits fractal character obeying power law with an exponent of 1.8 for the radius of gyration. At higher pressure the growth is more compact and dense and fractal dimension based on radius of gyration approaches 2, indicating non-fractal homogeneous growth. In the present paper Glycerol is used under low and intermediate pressure conditions the growth of fingering pattern is less compact as compared to at higher pressures. The shapes exhibit self similarity and scaling over wide range of distance. The fractal dimensions obtained using Box counting technique (T.A. Witten & Y Kantor

1984 and P. Meakin 1983). Under appropriate approximations the Laplace equation can express the interface, as in diffusion-limited process

$$\nabla^2 u(x,t) = 0 \qquad (1.1)$$

Experimental Details:

The Hele Shaw cell consists of two parallel glass plates separated by very small gap (a). The two plates were provided with collar of 25 mm along periphery to confine the fluid the lower plate was bigger than the upper plate to accommodate the upper plate. The spacing between the two plates can be adjusted using spacers of desired thickness. The hole of Diameter 2.5 mm was drilled at the center of the upper plate to inject air or water inside the cell. For different pressure conditions viscous fingering patterns were photographed using SLR camera with Zoom lens. We studied different fluids fingering pattern in figure (1A & 1B) typical fingering pattern of Glycerol at Low & High pressure is shown respectively. The photographs are digitized and converted to a matrix form for processing. The fractal dimensions obtained using box counting technique is presented. The shapes are also analyzed for structure and texture of the boundaries using Richardson's Plot technique.

Figure 1A is an open structure growing radially with increasing channel diameter is characteristic of phenomenon, the shape has limited structural has almost no fine texture. This yields fractal dimension of about 1.6. The box counting fractal dimension of pattern for high pressure, figure 1B, comes out to be 1.80. A plot of Log(N) Vs Log(r) is shown in figure 2, the points plotted are actual points and the line joining them is least square fit to the data point giving R^2 of 0.9996. The scaling exists over four orders as is seen from the graph. At high pressure the fingering obtained deviates from as shown in the figure 1A and tends to approach shapes as shown in figure 1B where the increase in the diameter of channel with increasing radius of the shape gradually disappears. At high pressure this effect simply disappears and effect is seen localized to small regions in the secondary branches only (G. M. Mitchell *et. al*. 2002). These fingering structures are very similar to pictures of 2D, DLA aggregates found in the copper growth (T.A. Witten & Y Kantor 1984 and P. Meakin 1983). They are tree like in a structure showing no loops. The fingers are narrow and only occasionally surround several spheres. By closer inspection one finds that only the outer fingers grow, the others are screened, as is the case for the growth of DLA cluster.

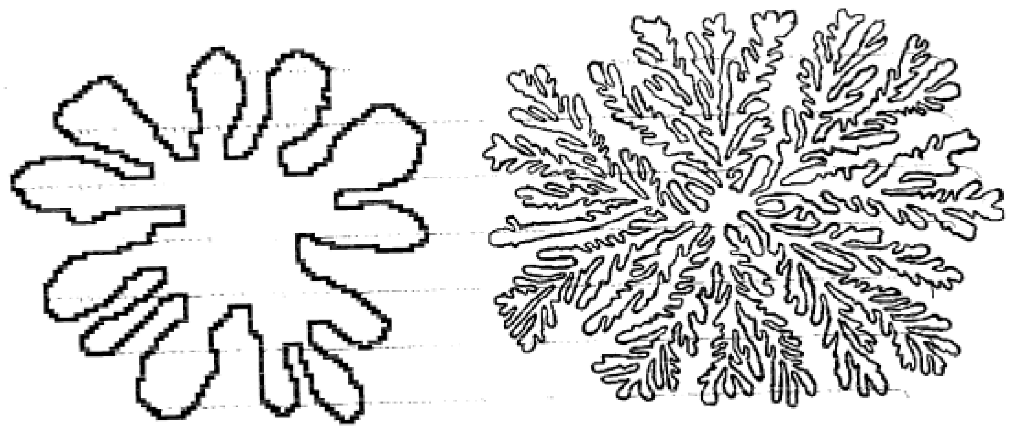

Figure 1. Viscous Fingering Patterns Obtained

A. With Low pressure B. With High Pressure.

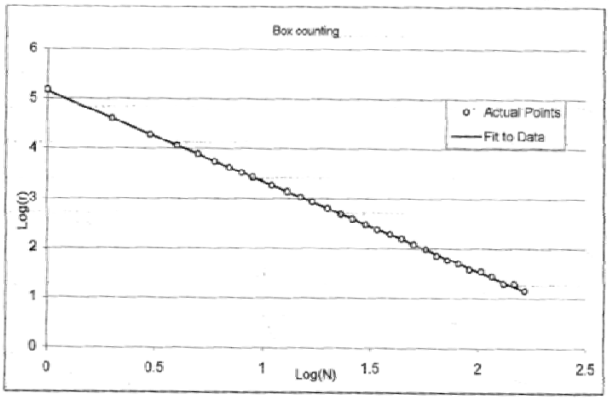

Fig: 2. Plot of Log(N) Vs. Log(r) for Fig 1 B

The relation of viscous fingering to Laplacian growth [5] can be shown by assuming that the plates are horizontal, and the flow in the x, y plane has a velocity profile

$v(z) = [v_x^2(z) + v_y^2(z)]^{1/2}$,

which is approximately parabolic in the direction z perpendicular to the plates.[10]

$$v(z) = a\left(\frac{b^2}{4} - z^2\right) \qquad (1.2)$$

Furthermore, we assume that $v_z = 0$ and $\frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial y} = 0$. For the average velocity one has

$$\bar{\vec{v}} = \frac{1}{b}\int_{-\frac{b}{2}}^{\frac{b}{2}} v(z)dz = \frac{ab^2}{6} \qquad (1.3)$$

If the gravitational effects can be neglected the Navier-Stockes equation has the form

$$\nabla p = \mu\nabla^2\vec{v} + \frac{\rho\partial\vec{v}}{\partial t} \qquad (1.4)$$

Where μ is the velocity of the fluid, ρ is the density, and p denotes the pressure. For small b the first term of the right-hand side in (1.3) dominates, because it is proportional to $1/b^2$. Inserting (1.2) into (1.4) (where the second term of the right-hand side is neglected) and using (1.3) we get

$$\bar{\vec{v}} = -\frac{b^2}{12\mu_1}\nabla p, \qquad \text{..........(1.5)}$$

The above equation represents the so called Darcy's law [5, 8] expressing the fact that for small b the average velocity is proportional to the local force. Assuming that the fluids are incompressible one arrives at the Laplace equation $\nabla^2 p = 0$ (1.1) for the pressure distribution p from the condition that the divergence of the velocity vanishes.

Discussion & Conclusion:

In summary we have shown that the viscous fingering at high pressure is similar to DLA clusters, being tree like in structure having no loops and D=1.62 (G. M. Mitchell *et. al*. 2002). While studying Viscous Fingering in a Hele Shaw type cell, we found that almost for all pressure ranges the pattern obtained exhibits the fractal character. Viscous fingering is studied using thick oil as more viscous fluid and air as less viscous fluid. It is observed that the fractal dimension at shorter length scale is less than that at longer length's scale. This shows that complexity of structure is greater as compared to the complexity of texture. There is a relation between operating pressure and complexity of structure and texture (P. G. Saffman 1958 and T.A. Witten et. al. 1984). Scaling observed at almost all the length scales. The box counting dimensional dimension gradually increases with increase in pressure. At low pressure the shape resembles more with the Laplacian growth pattern. At higher pressures the shapes deviate from the Laplacian growth pattern giving rise to branching and fine structure, the patterns are also dense with higher fractal dimensions. The phenomenon is also of practical importance in the recovery of oil.

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